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COMPUTATION OF MULTI-PLANE IMBALANCE FOR A MULTI-BEARING ROTOR SYSTEM

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1. Introduction

There have been a number of established procedures for balancing large rotating machines. Most of these procedures have been developed for balancing rotating machines prior to their operation. Many of them assume a linear model and require many test runs for the full identification of residual imbalance in the specified correction planes.

In the case of a turbogenerator set, changes in the balancing conditions may occur during its operation—this may be due to the result of the machine losing one or more of its blades. Development of procedures for on-site identification of imbalance changes has, therefore, drawn much attention. On-site identification involves using real-time vibration signals measured from a machine during its operation. Since the monitored dynamic information is often limited, proposals have been made to combine the efforts in measurements with the modelling and numerical analysis of the system to aid the balancing of a rotating machine such as a large turbine generator set.

In many situations, large amplitude vibrations may be from dramatic change in balancing conditions such as the loss of blades during the operation of a turbogenerator. The presence of large amplitude vibrations often implies a machine operating beyond the linearized equilibrium, particularly in the case of a rotating machine using oil bearings. The linearization of the system in these cases may lead to poor assessment of the identified parameters.

To handle large amplitude vibration problems, studies have been reported that focus on the non-linear dynamic characteristics of rotating machines. Krodkiewski *et al*. [1] presented a method that uses a non-linear mathematical model for the on-site identification of imbalance change that may take place during the operation of a multi-bearing rotor system. The mathematical model includes the dynamic properties of the rotor and foundations as well as the non-linearity of the oil bearings.

In brief, the method proposes that the signals measured before and after the imbalance change takes place are used to compute the time history of the hydrodynamic forces generated by the supporting journal bearings. Both the measured signals and the bearing forces are processed using a fast Fourier transform technique. The data obtained from this process are then combined with the physical properties of the system represented by mass and stiffness matrices to form a system equation. By using error functions and developing certain criteria, the location and the magnitude of imbalance change are identified.

The attractiveness of the method is that it requires only the relative journal-to-bearing displacements or velocities as input parameters to identify the change of imbalance. In modern rotating machinery, it is common practice to place permanent probes into the main supporting bearings as a means of "health monitoring" or "condition monitoring". The real-time vibration signals measured by these probes from a machine during its operation

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represent the journal-to-bearing displacements or velocities. Hence the real-time information required by the method is readily available in some practical situations.

The method proposed in reference [1] assumes, however, that the change in system responses is due to the change in imbalance at one plane of the rotor only (for example, a few blades are lost from one row of a turbogenerator set). Hence it was proposed to identify one location each time where an imbalance has taken place.

It is often desirable to identify in a round of computation several locations. This paper presents a further development of the above method and provides the basis for the identification of more locations.

2. THE MATHEMATICAL MODEL

A non-linear mathematical model for multi-bearing rotor systems is given in references [1, 2]. For the convenience of discussion, the equation of motion is written as follows,

$$\mathbf{M}\ddot{\mathbf{r}} + \mathbf{K}\mathbf{r} = \mathbf{H} + \mathbf{P} + \mathbf{U}\,\mathrm{e}^{\mathrm{i}\Omega t} \tag{1}$$

The displacement vector $\mathbf{r}$ determines the instantaneous positions of the stations on the rotor with respect to the absolute system of co-ordinates XYZ as shown in Figure 1.

The dynamic properties of the rotor are determined by the mass matrix $\mathbf{M}$ and the stiffness matrix $\mathbf{K}$. These matrices may be obtained by using a finite element model for the rotor in consideration or by some experimental means. Since it is often desirable to eliminate the unwanted degrees of freedom resulting from the finite element analysis, a method by Guyan can be used to accomplish this task. Alternatives include a component mode method by Craggs [3] and many other methods [4–7] aiming at the reduction of excessive unknowns.

The interaction between the oil bearings and the rotor is represented by the vector $\mathbf{H}$. This vector has a total number of $M + N$ elements, where M is the number of supported stations or the number of bearings and N is the number of unsupported stations or the the number of balancing planes, i.e.,

$$\mathbf{H} = (\mathbf{H}_1, \mathbf{H}_2, \ldots, \mathbf{H}_M, 0, \ldots, 0)^{\mathrm{T}} \tag{2}$$

The bearing forces are a non-linear function of the journal-to-bearing relative displacements, relative velocities and the rotating speed of the rotor. The static load is denoted by $\mathbf{P}$ which includes the dead load of the rotor itself and all the other static loads imposed on the system. This static load cannot be omitted here in the dynamic analysis because the model is a non-linear one. The non-linearity of the system lies in the non-linear interaction force vector $\mathbf{H}$. The static load determines the system's equilibrium position in the case of stable steady state operation. The imbalance forces are denoted by the vector

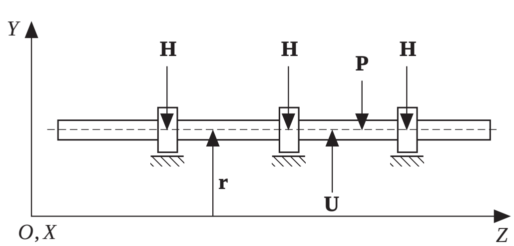

Figure 1. Co-ordinate system for a multi-bearing rotor system, where **H** are the non-linear dynamic hydraulic forces, **P** the static loads such as the weight of the rotor etc., **U** the dynamic imbalance forces and **r** the absolute physical co-ordinates.

U. In the above mathematical model there is no damping matrix because the damping and velocity dependent forces are allowed for in the non-linear interaction force **H**.

3. THE METHOD

An analysis of the above mathematical model has resulted in the following equation (see reference [1]):

$$\mathbf{Cu} = \mathbf{s} \tag{3}$$

where **C** is a rectangular coefficient matrix, **u** is an unknown vector containing the imbalance changes at the unsupported stations and **s** is a vector whose dimension is equal to the number of supported stations. Details of the analysis are given in reference [1] where an algorithm is presented for the identification of imbalance change at one plane.

It is noted that the element of matrix **C** and vector **s** are supposed to be known for the purpose of identifying the imbalance changes. As a matter of fact, they are dependent purely on the monitored system responses and the predicted system parameters such as the mass and stiffness matrices.

This paper focuses on the discussion of the above equation and presents a method for the identification of imbalance changes at two planes.

According to reference [1], the full expansion of the above equation is written as:

$$\begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1p} & \cdots & c_{1q} & \cdots & c_{1N} \\ c_{21} & c_{22} & \cdots & c_{2p} & \cdots & c_{2q} & \cdots & c_{2N} \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ c_{i1} & c_{i2} & \cdots & c_{ip} & \cdots & c_{iq} & \cdots & c_{iN} \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ c_{M1} & c_{M2} & \cdots & c_{Mp} & \cdots & c_{Mq} & \cdots & c_{MN} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \vdots \\ \mu_p \\ \vdots \\ \mu_q \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} s_1 \\ s_2 \\ \vdots \\ s_i \\ \vdots \\ s_M \end{bmatrix} \tag{4}$$

In the above equation, M represents the number of supported stations (or the number of supporting bearings), N the number of unsupported stations, and p and q indicate the two unspecified planes (or stations) where the imbalance changes take place.

Also it is assumed in the above equation that imbalance changes take place only at two planes. This assumption is made here only for the convenience of discussion. The situation for more than two planes can be handled when the problem for two-plane is solved.

As is seen, the column matrix contains only two elements that have positive non-zero values. All the rest are zeros. As illustrated in Figure 2, μ_p and μ_q are positive because they represent the magnitudes of imbalance change taking place at the two stations p and q. The phase of the imbalance change is accounted for by the angle between the two vectors $\mathbf{U}_{before}$ and $\mathbf{U}_{after}$ (refer to Figure 2). By expanding the above matrix equation one obtains the following algebraic equations,

$$\begin{gathered} c_{1p}\mu_p + c_{1q}\mu_q = s_1 \\ c_{2p}\mu_p + c_{2q}\mu_q = s_2 \\ \cdots\cdots \\ c_{Mp}\mu_p + c_{Mq}\mu_q = s_M \end{gathered} \tag{5}$$

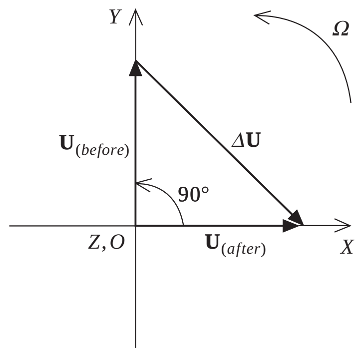

Figure 2. Illustration of an imbalance change.

where p and q each may take any one number among $(1, 2, \ldots, N)$ but $p \neq q$.

If a system has exactly two supported stations (for example, a two-bearing rotating machine), then M equals 2. This means that there are exactly two equations that can be used to solve the two unknowns—μ_p and μ_q. In this case, the problem becomes straightforward.

If a system has more than two supported stations (for example, a large turbogenerator unit), then M is greater than 2. In this case, the number of equations is more than that of the unknowns. Any two of the above equations, however, may be used to compute the imbalance changes, and all the rest of the equations must be unconditionally fulfilled.

By rearrangement, the above set of equations becomes:

$$(c_{ip}/s_i)\alpha + c_{iq}/s_i = 1/\mu_q, \qquad i = 1, 2, \ldots, M \tag{6}$$

where $\alpha = \mu_p/\mu_q$. The factor α is the ratio of the imbalance changes. As far as the above set of equations is concerned, α is a positive number.

In all the above equations, the right side is the same. By eliminating this factor between any two of the equations, one obtains the following:

$$(c_{ip}/s_i)\alpha + c_{iq}/s_i = (c_{jp}/s_j)\alpha + c_{jq}/s_j, \qquad i = 1, 2, \ldots, M, \quad j = 1, 2, \ldots, M \tag{7}$$

Numerical computations will hardly result in an absolute equation as the above ones. This means that if the left side is subtracted by the right side the result may not be exactly zero. Rather it may be a small number. If one uses ε to represent this small number, then it becomes:

$$\varepsilon_{ij} = \frac{c_{ip}}{s_i}\alpha + \frac{c_{iq}}{s_i} - \left(\frac{c_{jp}}{s_j}\alpha + \frac{c_{jq}}{s_j}\right) = \left(\frac{c_{ip}}{s_i} - \frac{c_{jp}}{s_j}\right)\alpha + \left(\frac{c_{iq}}{s_i} - \frac{c_{jq}}{s_j}\right); \quad \begin{array}{l} i = 1, 2, \ldots, M \\ j = 1, 2, \ldots, M \end{array} \tag{8}$$

An error function in this case can be defined as:

$$E_{pq}(\alpha) = \frac{1}{M^2} \sum_{j=1}^{M} \sum_{i=1}^{M} \varepsilon_{ij}^2 \tag{9}$$

Note that this is the error function E corresponding to the assumed p and q. As was stated previously, if the planes p and q are truly the planes where the imbalance changes take place, then the error function E must be zero in theory. In practice, it must be very small compared with the other evaluated error functions corresponding to the combinations of any two planes in the system. On the other hand, the error function is also a function of the unknown ratio α. This means that, for different values of α, the value of error function may be different. One may start with a given value of α—say, 0·8, 1 or 10, etc.,—and repeat for the given α the process of evaluating all the error functions. If this process is chosen, then it is a second round of using the "trial and error" method. It may or may not lead to the correct identification of the imbalance change locations.

4. EVALUATION OF ERROR FUNCTIONS

It is advantageous to take into consideration the fact that the identification of imbalance changes relies on whether a smallest error function can be found. Instead of arbitrarily giving a value for the α one may take the advantage of differentiating the error function with respect to the α. The differentiation gives,

$$\frac{\mathrm{d}E_{pq}}{\mathrm{d}\alpha} = \frac{2}{M^2}\sum_{j=1}^{M}\sum_{i=1}^{M}\left\{\left(\frac{c_{ip}}{s_i}-\frac{c_{jp}}{s_j}\right)\alpha + \left(\frac{c_{iq}}{s_i}-\frac{c_{jq}}{s_j}\right)\left(\frac{c_{ip}}{s_i}-\frac{c_{jp}}{s_j}\right)\right\} \tag{10}$$

By equating the above derivative to 0 and solving for α, one obtains the following:

$$\alpha = \sum_{j=1}^{M}\sum_{i=1}^{M}\left(\frac{c_{iq}}{s_i}-\frac{c_{jq}}{s_j}\right)\left(\frac{c_{ip}}{s_i}-\frac{c_{jp}}{s_j}\right)\Bigg/\sum_{j=1}^{M}\sum_{i=1}^{M}\left(\frac{c_{ip}}{s_i}-\frac{c_{jp}}{s_j}\right) \tag{11}$$

With the value of α being introduced into the error function, it follows

$$E_{pq} = \frac{1}{M^2}\sum_{j=1}^{M}\sum_{i=1}^{M}\left(\left(\frac{c_{ip}}{s_i}-\frac{c_{jp}}{s_j}\right)\alpha + \left(\frac{c_{iq}}{s_i}-\frac{c_{jq}}{s_j}\right)\right)^2, \qquad \begin{array}{l} p = 1, 2, \ldots, N-1 \\ q = p+1, \ldots, N \end{array} \tag{12}$$

This is the equation for evaluating the error functions corresponding to all possible combinations of any two planes p and q among the N unsupported stations. It is easy to see that the number of error functions is K which is expressed as

$$K = (N^2 - N)/2 \tag{13}$$

There may be K different values for the constant α resulting from the differentiation of the K error functions. It is noteworthy that, when using the above method to determine the locations p and q, one may need to evaluate the error functions corresponding to some or all of the K values of α.

By theory, the error function corresponding to the two planes at which imbalance changes have taken place must be zero when evaluated using the above expressions. In a practical situation, however, it is rarely zero because there are always errors in measurements, numerical computation and discrepancies in mathematical modeling. In consideration of this fact, the following criterion can be adopted:

If there is an error function E_{pq} that is very small compared with the remaining error functions, then p and q must be the planes where imbalance changes have taken place.

With the locations being identified, it is not difficult to find the magnitude of the imbalance changes. There are a number of different methods for computing μ_p and μ_q. If the least square method is used, then the magnitude of the imbalance changes is given by:

$$\mu_q = \sum_{i=1}^{M}(c_{ip}\alpha + c_{iq})s_i \Bigg/ \sum_{i=1}^{M}(c_{ip}\alpha + c_{iq})^2, \qquad \mu_p = \alpha\mu_q \tag{14}$$

where the ratio between the imbalance magnitudes is given by equation (11).

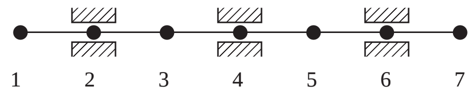

Figure 3. A flexible rotating shaft with three supports.

5. VERIFICATION

The proposed procedure was verified by means of computer simulation of a flexible rotating shaft with three supports. The system is schematically shown in Figure 3. The shaft was made of steel with a length of 2000 nm and uniform diameter of 35 mm. It was modelled using finite element method and condensed into 7 stations (or planes). As is seen from the illustration, the supports are located at stations 2, 4 and 6. The unsupported planes are 1, 3, 5, and 7. The three supports are identical journal bearings. Reynolds equations were used to model the dynamic characteristics of the journal bearings. By using a finite difference method to solve the Reynolds equations, the time history of the hydrodynamic forces were obtained.

The predicted physical properties of the rotor shaft and the dynamic characteristics of the journal bearings were integrated into a non-linear simulation system for the purpose of simulating the vibration responses of rotating machines under certain condition of excitations. This computer simulation system was based on the mathematical model mentioned above and discussed in detail in reference [2].

The system responses were simulated for 7 cases of imbalance excitation with a rotating speed of 3000 revolutions per minute (RPM). The imbalance distribution in case 1 was considered as the initial residual imbalance of the rotor system. The imbalance excitations specified in the rest of the cases were imposed to form the changes of imbalance with respect to the first case. Table 1 lists the differences in imbalance between case 1 and the other 6 cases.

For instance, the phase of the imbalance force at station 1 was changed from 90 degree in case 1 to 0 in case 2. As the initial amplitude of the imbalance force at station 1 is 5 N, such a change in the phase is equivalent to a blade loss which results in an increment of the imbalance force at station 1 by the amplitude of 7·07 N. This is shown in Figure 2.

Table 2 lists the evaluated error functions for the four unsupported stations for each of the six cases studied. It is clearly seen that the error function corresponding to the two stations where imbalance changes are imposed is much smaller than any other error function in the roll.

TABLE 1

Imbalance distribution for non-linear simulation of rotor vibration

	Station							
Case	Amplitude (N)	Phase	Amplitude (N)	Phase	Amplitude (N)	Phase	Amplitude (N)	Phase
1	5	90	2	0	4	60	3	30
2	–	0	–	–	15	–	–	–
3	–	–	–	45	–	–	20	–
4	15	–	15	–	–	–	–	–
5	–	–	–	–	20	–	15	0
6	0	–	–	–	–	–	10	–
7	–	–	20	–	15	–	–	–

Table 2

Error functions for the combination of any two unsupported stations

Case	Error function(%)					
	E_{13}	E_{15}	E_{17}	E_{35}	E_{37}	E_{57}
2	21·89	**0·45**	28·66	258·33	892·02	2568·46
3	899·65	680·21	165·33	30·24	**0·56**	50·25
4	**0·35**	43·98	256·36	569·46	786·45	1865·48
5	3569·25	456·45	298·47	56·21	20·45	**0·70**
6	298·00	23·78	**0·60**	56·82	456·12	894·20
7	982·03	465·23	89·78	**0·95**	32·05	145·68

In case 2, for instance, imbalance changes were imposed at stations 1 and 5. The corresponding error function E_{15} is 0·45% (the first row in Table 2), while all of the other error functions are at least 21·89% (the largest being 2568.46%). In case 3, imbalance changes were imposed at stations 3 and 7 and the corresponding error function E_{37} is 0·56%. The smallest error function in each case is used to locate the two stations at which imbalance changes were imposed.

A comparison between the imposed imbalance changes and the identified results is given in Table 3. It can be seen that there is a good agreement between the imposed and identified magnitude of imbalance changes. The largest difference (error) in any case is 1·2% while the smallest is only 0·4% (or −0·4%).

6. Summary and conclusions

The method presented in this paper is a further development of the one discussed in reference [1] where a consideration was given to the possibility of identifying imbalance change at one plane. The new method allows the identification of two planes where imbalance changes take place. It also provides a basis for the formation of algorithm for identifying multi-plane imbalance changes. These methods are devised for on-site identification purposes. The significance lies in that they make use of the relative journal-to-bearing motion signals only. These signals can be obtained by using the build-in transducers in the bearing housings. This is important for application to the identification of faulty unit of a turbogenerator set that has lost one or more of its blades.

Table 3

Identified stations, identified magnitude of imbalance change and the difference between the identified and imposed magnitude

Case	Station	Magnitude of imbalance change			Station	Magnitude of imbalance change		
		Identified (N)	Original (N)	Error (%)		Identified (N)	Original (N)	Error (%)
2	1	7·11	7·07	0·5	5	11·07	11·00	0·6
3	3	1·54	1·53	0·7	7	16·88	17·00	−0·7
4	1	10·04	10·00	0·4	3	12·95	13·00	−0·4
5	5	15·90	16·00	−0·6	7	12·64	12·49	1·2
6	1	4·96	5·00	−0·9	7	6·95	7·00	−0·7
7	3	18·20	18·00	1·1	5	11·05	11·00	0·5

Verification of the method was conducted using the computer simulation of a three-support rotor system. The computer program was used to simulate the dynamic responses of the rotor system under imposed imbalance conditions. The journal-to-bearing displacements were taken and processed according to the proposed algorithm in order to identify system imbalance changes in different cases. For these cases considered, comparison of the identified results with the pre-imposed imbalance conditions shows that the method is theoretically reliable.

Additional investigation will be required in order to assess the sensitivity of the method to other parameters such as the noise level present in the motion signals and the predicted system physical properties. Finally, comprehensive experimental verifications will have to be conducted in order to confirm the applicability of the method.

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